

# Math 9

# POLYNOMIALS

## Chapter 4 Package

The diagram shows the polynomial  $5x^2 + 2y - 7$  with color-coded parts and labels connected by lines:

- Coefficient** (purple) points to the  $5$  in  $5x^2$ .
- Exponent** (blue) points to the  $2$  in  $x^2$ .
- Variable** (green) points to the  $x$  in  $x^2$  and the  $y$  in  $2y$ .
- Operator** (grey) points to the  $+$  and  $-$  signs.
- Constant** (pink) points to the  $7$  in  $-7$ .

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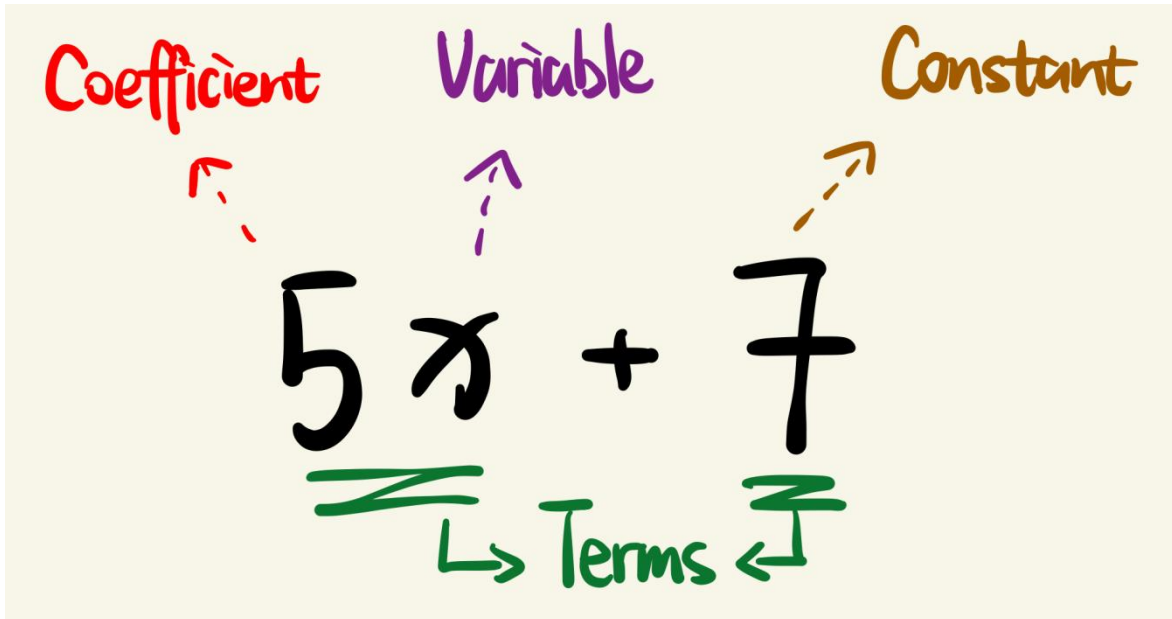
NAME: \_\_\_\_\_

Frayer Model Template

Definition	<div>Variable</div>		Facts/Characteristic	Definition	<div>Coefficient</div>		Facts/Characteristic
Example			Non-Example	Example			Non-Example
Definition	<div>Constant</div>		Facts/Characteristic	Definition	<div>Term</div>		Facts/Characteristic
Example			Non-Example	Example			Non-Example

# Lesson 1: Introduction to Polynomials

## - Terms



**Example 1:** Complete the chart

Term	Coefficient	Variable
$25x$		
$-4.9t^2$		
$xy$		
$2.5$		

## What is Algebra?

➤ **Algebra** is a \_\_\_\_\_ of mathematics that uses \_\_\_\_\_ to represent unknown numbers or \_\_\_\_\_.

➤ An **Algebraic Expression** is a mathematical \_\_\_\_\_ that combines \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_.

HAS NO \_\_\_\_\_ SIGN!

Ex)  $5x + 8$

## What is Polynomials?

➤ A **Polynomial** is an \_\_\_\_\_ expression made up of \_\_\_\_\_ connected by an \_\_\_\_\_.

Ex)  $2x + 3$

➤ A **Polynomial** can have:

✓ Constants

(Example: \_\_\_\_\_)

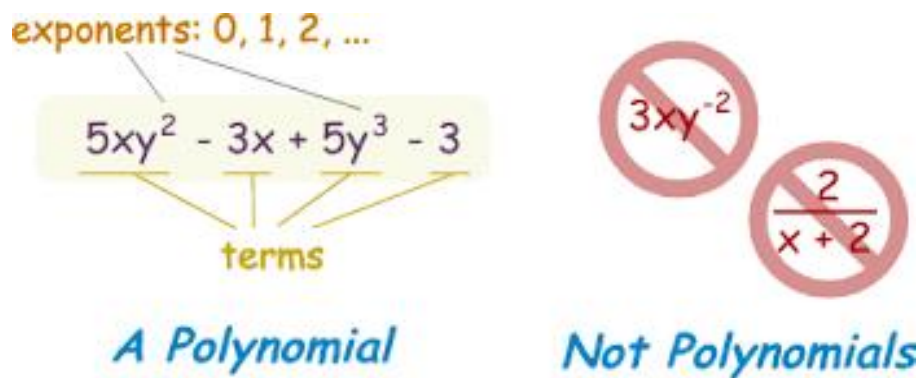
✓ Variables

(Example: \_\_\_\_\_)

✓ Exponents

(Example: \_\_\_\_\_)

- ✧ Polynomials can't have variables that are being divided, have negative exponents, or square roots.



➤ There are three main types of polynomials:

**Monomials:** A polynomial with \_\_\_\_\_ term. Ex)

**Binomials:** A polynomial with \_\_\_\_\_ terms. Ex)

**Trinomials:** A polynomial with \_\_\_\_\_ terms. Ex)

Polynomial: \_\_\_\_\_ or more terms.

**Example 2:** Classify each Polynomial by the number of terms.

Expression	Number of terms	Name
a) $3 + m^2$		
b) $ab^2 + ab - 1$		
c) $5x^2$		
d) $-4x^2 + xy + y^2 - 10$		

# Lesson 2: Introduction to Polynomials

## - Degree

- The **Degree of a Term** is the sum of the \_\_\_\_\_ on the variables in a \_\_\_\_\_ term.

Ex)  $2x^2$

Ex)  $2yx^2$

- The **Degree of a Polynomial** is the \_\_\_\_\_ degree of any of the \_\_\_\_\_ in a polynomial.

**Example:**

a)  $7a^3 - 3a$

b)  $3x^4y^2 - 54x^3y + 12x^2 + 6x - 34$

## ✧ Practice - Terms & Degree

**Practice 1:** Identify the Number of Terms, Degree, and Name of each Polynomial.

Expression	Number of terms	Degree	Name
a) $1 + 3x$			
b) $4x + 3xy - 7$			
c) $-27b^2$			
d) 99			
e) $x^3 + 2xy - 3xyz$			

**Practice 2:** Evaluate Polynomials (using BEDMAS)

✧ Recall that the variables in a polynomial represent unknown numbers.

a)  $x - 1$  when  $x = -3$

b)  $x^2 + x - 2$  when  $x = -2$

c)  $8 - 2(x + 2y)$  when  $x = -5$  and  $y = -1$

d) Evaluate the following expression if  $x = 2$  and  $y = 3$ . Show how you simplify.

$$4x^2y + (x - 1)(y + 5) - 10$$

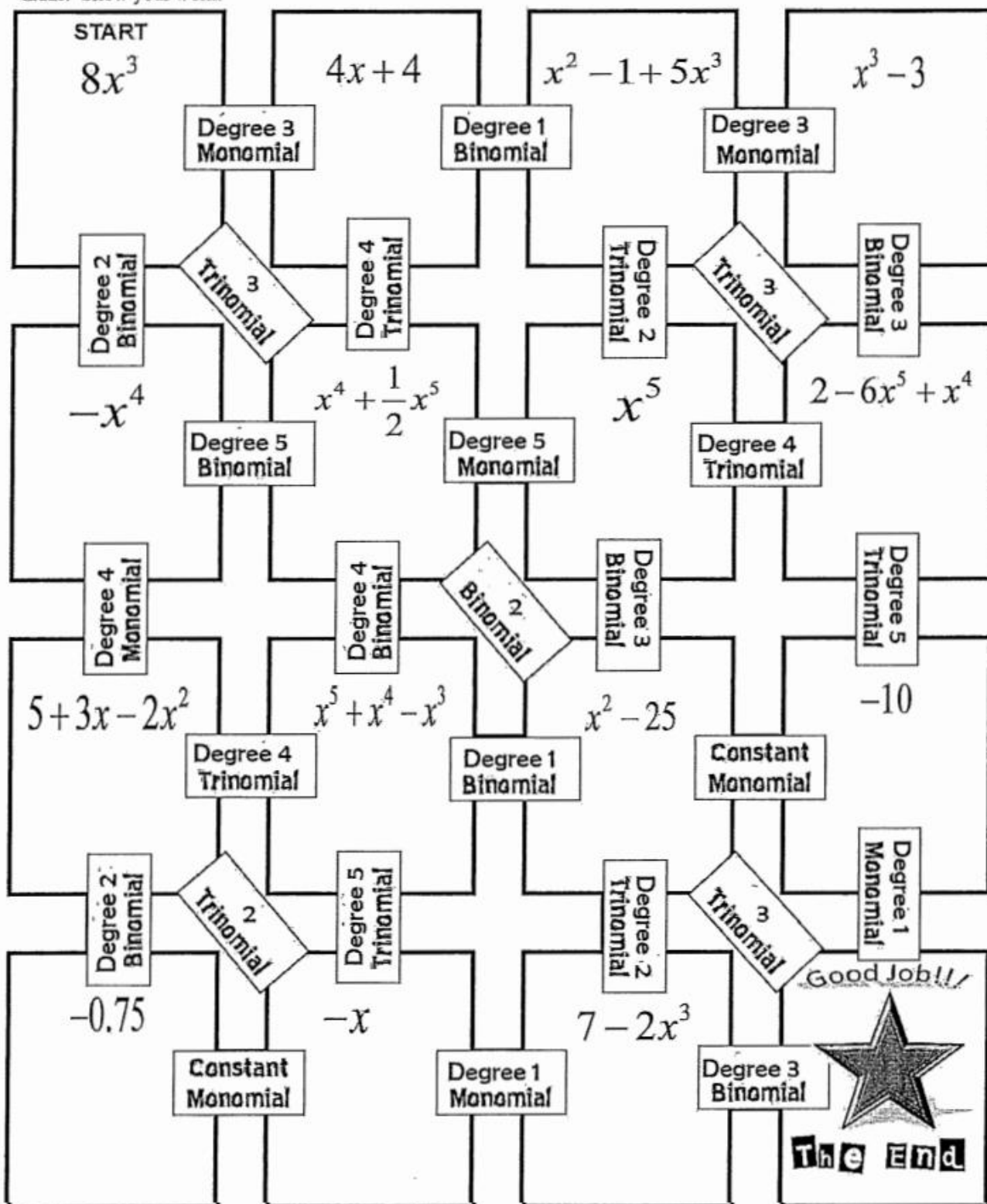
e) Evaluate the following expression if  $p = 3$  and  $q = 1$ . Show how you simplify using.

$$p^4 + 4p^3q + 6q^2q^2 + 4pq^3 + q^4$$



# Classifying Polynomials Maze!!!

**Directions:** Classify each polynomial by degree and by number of terms. Use your solution to navigate through the maze. Show your work.



# K-W-L Chart

Topic: \_\_\_\_\_

**K**

What I Know:

**W**

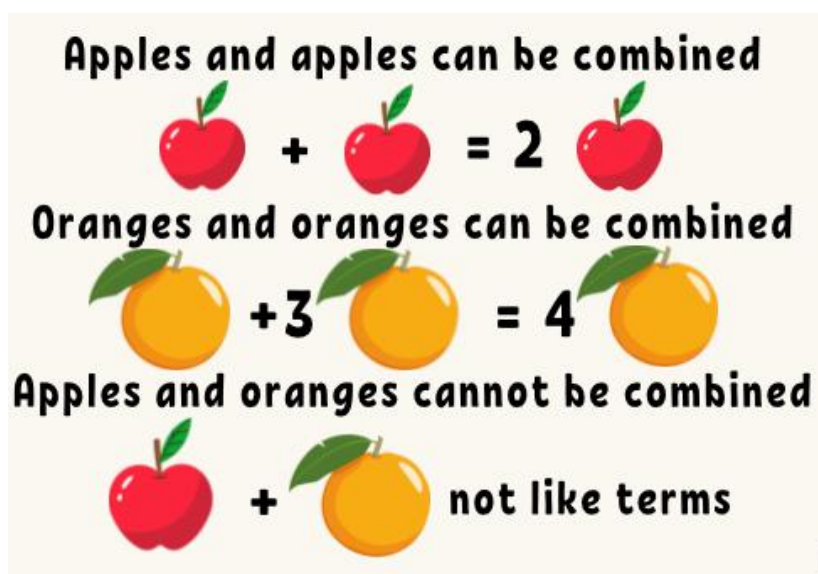
What I Want to Know:

**L**

What I Have Learned:

## Lesson 3: Like Terms

- ✧ Like terms are just like objects with common feature.



- ✧ Like terms have same variables to the same power.

$$\begin{array}{l} 3a \quad 12a \quad 54a \\ 5x \quad 12x \quad 0.9x \\ 3xy^2 \quad 9xy^2 \quad 12xy^2 \end{array}$$

- ✧ Use shape to identify like terms.

Example:  $5a + 2x + 8ab + 9b + 12ab + 7b + 3x + 10a + 4ab$

**Practice: Which in each group are like terms?**

a)  $2b$        $-2a$        $5$        $6b$        $-3a^2$

b)  $3pq^2$        $-0.4q$        $-1.2p$        $3p^2q$        $-1.2pq^2$

c)  $2a^2$        $a$        $5b^2$        $b$        $-3.2b^2$

d)  $-xy$        $-3x^2$        $7xy$        $3y$        $1.4xy$

**Combining Like Terms:**

✧ You can combine like terms to simplify the algebraic expressions by adding/subtracting the coefficients.

1)  $3x + 5x$

2)  $5k - 2k$

3)  $b + b + c + c$

## ✧ Practice - Like Terms

1. Answer the questions or identify the specified parts of the polynomial:

$$5x^3 - 2x^2 + 14x + 7x^2 + 3x - 11$$

- a) How many terms does this polynomial have? \_\_\_\_\_
- b) Write the term that has a degree of 3. \_\_\_\_\_
- c) What is the coefficient of this term with degree 3? \_\_\_\_\_
- d) Which term is a constant term? \_\_\_\_\_
- e) List a pair of like terms. \_\_\_\_\_ and \_\_\_\_\_
- f) List another pair of like terms. \_\_\_\_\_ and \_\_\_\_\_
- g) Rewrite the polynomial in simplified form by combining the like terms:
  
  
  
  
  
  
  
  
  
- h) Evaluate this polynomial for  $x = 2$ . \_\_\_\_\_

2. Can you simplify the expressions by combining the like terms?

- a)  $3b - b + 4b$

b)  $4b + 2b + 3a + 2a$

c)  $-7y + 8x - 9x - y$

d)  $5xy + 6xy + 10 - 3$

e)  $-3a + b + 5ab - 8a - 3ab + 2b$

f)  $3x^2 + y^2 - x + 2x^2 - 4x + 5y^2 + 10$

g)  $4rs^2 - 6r^2s + 3rs - 3r^2s + rs^2$

**Challenge:** Can you try to simplify the expressions?

a)  $-2w^5 + 5w^3 - w^2 + 11w^4 - 8w^3 + 2 + 2w^2 - w^5 - 9$

b)  $7y^2 + 13xy - 2x^2 + y^2 - 11 - 8xy + 3x^2 - 4 - xy - y^2$

c)  $2m^2n + 10mn^2 + 8n^2 - 5mn - 16 + 3nm - m^2 + 13 + 5nm^2 - 3mn^2 - 2m^2 + n^2 + 1$

# Lesson 4: Adding and Subtracting Polynomials

To **ADD** Polynomials:

**Step 1:** Remove the \_\_\_\_\_ from each polynomial.

**Step 2:** \_\_\_\_\_ the equation so that \_\_\_\_\_ are together.

**Step 3:** \_\_\_\_\_ like terms.

**Example 1:** Add the following polynomials.

A)  $(5x + 1) + (3x + 2)$

B)  $(2x^2 - 4x - 1) + (3x^2 + 2x + 5)$

C)  $(x^2 - 5x + 6) + (10x^2 - 3x + 14)$

D)  $(4x^4 + 2x^3 - 3x^2 - 5x + 9) + (-5x^3 - 6x^2 + 2x - 4)$



To **SUBTRACT** Polynomials:

**Step 1:** The subtraction is affecting everything inside the following bracket.

So we must bring it to every term in the following bracket **AND** drop the brackets

**Step 2:** Repeat **ADD** steps as above.

**Example 2:** Subtract the following polynomials.

a)  $(5x + 4) - (2x + 1)$

b)  $(x - 3) - (6x - 5)$

c)  $(3x^2 + 4x - 2) - (2x^2 + 6x + 2)$

d)  $(2x^3 - 5x + 3) - (-5x^3 - 4x^2 + x - 1)$

### ✧ Column Form Method

Adding these two polynomials:

$$(3x^2 + 5x - 4) + (x^2 - 9x + 7)$$

Using column form:

$$\begin{array}{r} 3x^2 + 5x - 4 \\ + \quad x^2 - 9x + 7 \\ \hline 4x^2 - 4x + 3 \end{array}$$

Get the answer:

$$4x^2 - 4x + 3$$

**Example 3:** Combining the following polynomials

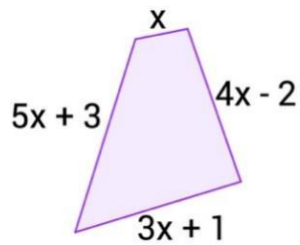
$$(2x^2 - 4x - 1) + (3x^2 + 2x + 5)$$

$$(3x^2 + 4x - 2) - (2x^2 + 6x + 2)$$

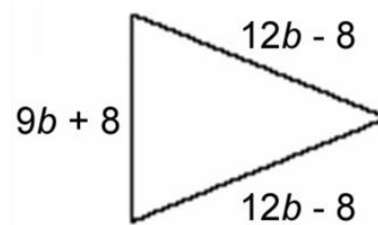
✧ Combining more than 2 polynomials

**Example 4:** Miss. Ma and Mrs. Isaak both have small gardens and they need to acquire some fences to enclose their gardens. The length of each side of each garden is as follows, so please help them to work out how long each fence will need to be. (Unit: meters)

*(Hint: for perimeter you just add up all the outside lengths. )*



Mrs. Isaak's Garden



Miss. Ma's Garden

## ✧ Practice - Adding/Subtracting Polynomials

1. Add the following polynomials:

a)  $(6x + 2) + (3x + 4)$

b)  $(-x + 4) + (7x - 2)$

c)  $(5a - 3) + (2a + 7)$

d)  $(-1 - 3t) + (4 - 5t)$

e)  $(8 - 4m) + (-3 - 2m)$

f)  $(9c - 2) + (-5c - 3)$

g)  $(3x + 4) + (5x + 2) + 2x$

h)  $(-2x^2 + 6x - 7) + (3x^2 - x - 2)$

i)  $(3x + 5) + (2x - 3) + (4x - 6)$

j)  $(5x - 3) + (4x + 7) + (2x - 6)$

2. Subtract the following polynomials:

a)  $(-2x + 3) - (3x + 2)$

b)  $(1 - 3t) - (-2 - 5t)$

c)  $(4 - 5n) - (-6n + 2)$

d)  $(3a - 5) - (6 - 7a)$

e)  $(7x - 25) - (17 + 5x)$

f)  $(5x^2 - 6x + 1) - (4x^2 - 2x + 5)$

g)  $(8a^2 + 2a - 3) - (-6a^2 + 4a + 7)$

h)  $(7c - 5) - (-c + 3) - (2c - 1)$

3. Add/subtract the following polynomials:

a)  $(3x - 2) - (x - 1) + (4x - 3)$

b)  $(2a + 3) + (6a - 1) - (a - 5)$

c)  $(4x^2 - 3x) - (x^2 + 2x) + (3x^2 - x)$

d)  $(2m^2 - 5) + (3m - 2) - (m^2 + 1)$

e)  $(5t - 4) + (3t^2 - t) - (2t + t^2)$

f)  $(17x - 25) + (34x + 19) - (23x - 11)$

g)  $(45 - 10x) - (-15 - 25x) - (35x + 10)$

h)  $(25n^2 - 6) - (30n^2 - 2n) + (5n^2 + 3n)$

i)  $(17x - 25) + (34x + 19) - (23x - 11)$

j)  $(37 - 42t) - (61 + 23t) + (21 - 17t)$

## Lesson 5: Multiplying Polynomials by Monomials

**Example 1:** Compare adding and multiplying.

a)  $6 + 6$  vs  $6 \times 6$

b)  $x + x$  vs  $x \times x$

c)  $3x + 5x$  vs  $3x \times 5x$

### ✧ Multiplying Polynomials Using Exponent Law

If the variable is the same but has different exponents of the given polynomials, then we need to use the exponent law.

**Example 2:** Multiply  $2x^2 \times 3x$

Here, the coefficients and variables are multiplied separately.

$$= (2 \times 3) \times (x^2 \times x)$$

$$= 6 \times x^{2+1}$$

$$= 6x^3$$

### ✧ Monomial $\times$ Monomial

Multiply the coefficients and add the exponents!

Example 3:

a)  $(2x)(4x)$

b)  $(-6z^2v)(-2z^3v)$

### ✧ Monomial $\times$ Polynomial

In order to multiply polynomials by monomials, we must use something called the **Distributive Property**.

Because the monomial is multiplying \_\_\_\_\_ in the following polynomial.

Example 4:

a)  $4(3x-5)$

b)  $-x^2(4x + 2)$

c)  $-7y(2x - 4y)$

d)  $2x(6x^2 + 3x - 1)$



**Example 5:** State whether the two expressions are equivalent or not equivalent.

a)  $3(x^2 + 4x)$  and  $3x(x + 4)$

b)  $-5x(6x - 5)$  and  $x(25 - 30x)$

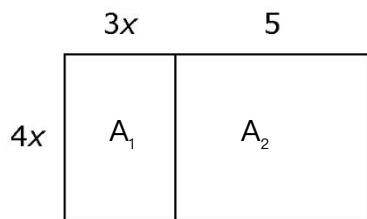
**Example 6:** Expand using the Distributive Property and simplify.

a)  $5r^3(2r - 4) + 4r(3r^3 - r) - 2r^2(6 - 8r^2)$

**Example 7:** Solve

Calculate the area of each rectangle:

$$A_1 =$$



$$A_2 =$$

The total area =

## ✧Practice - Multiplying Polynomials

1. Multiply then simplify the following polynomials:

a)  $(3a^3)(2a)$

b)  $5(x - 3)$

c)  $7(a + 1)$

d)  $-3(2 + n)$

e)  $-1(2x - 5)$

f)  $-4(x - 2)$

g)  $x^2(3x - 1)$

h)  $a(5a - 1)$

i)  $5(x^2 - 6x + 3)$

j)  $-3x(2x + 3y - 5)$

k)  $-2(-3 + 5n - 3n^2)$

l)  $n^2(3n^2 - 5n + 1)$

m)  $3(2a - 4b) + 5(3a + 2b)$

n)  $2x(x - 3) - (x^2 - 5)$

o)  $y^3(y - 5) + 6$

p)  $\frac{1}{2}(4m - 6) - \frac{2}{3}(9m + 3)$

q)  $\frac{3}{4}(8x - 12) + \frac{1}{3}(18x - 9)$

## Lesson 6: Dividing Polynomials by Monomials

✧ Monomials  $\div$  Monomials:

a)  $x^5 \div x^3$

b)  $8x^{12} \div 2x^5$

c)  $\frac{15d^2r^4}{3d^2r^3}$

✧ Polynomials  $\div$  Monomials:

- When dividing a polynomial by a monomial: The denominator is being divided by every term in the numerator.

Example:

a)  $\frac{6x + 9}{3}$

b)  $\frac{20x^2 - 18x}{4x}$

$$\text{c) } \frac{6k^2 + 12k + 3}{3}$$

$$\text{d) } \frac{28r^5 - 22r^4 + 20r^3 - 18r^2 + 12r}{6r}$$

## ✧ Practice - Dividing Polynomials

1. Divide then simplify the following polynomials:

a)  $\frac{4r}{2}$

b)  $\frac{15x}{3x}$

c)  $\frac{2a^4b^3}{a^2b}$

d)  $\frac{20d^5e^3f^5}{12d^2e^3f^4}$

e)  $\frac{6x+3}{3}$

f)  $\frac{-st-8t}{-2t}$

g)  $\frac{14k^2 - 21k}{7k}$

h)  $\frac{12c^2 - 15bc + 18c}{3c}$

i)  $\frac{15k^2 - 12jk + 27k}{-3k}$

j)  $\frac{12x^6y^4 + 6x^4y^3 + 3x^3y^2}{-3x^3y^2}$